

Coin Tossing Introduction

In this activity, students should have had some experience with the notion of probability as an expression of the likelihood of an event occurring given a "fair test". In addition they will need to have developed some facility with finding the equivalencies between fractions and percents and representing data using fractions.

This activity will develop two key ideas in the mathematics of probability:

- relating the difference between expected (or theoretical probability) and experimental (or actual probability);
- realizing that when the number of actual events increases the experimental outcome will tend to get closer to the theoretical outcome.





Discussion Guide

Open the discussion by asking the children to give familiar examples of categories of repeated events that have a number of fixed possible outcomes. Examples might include, rolling dice or spinning a wheel in a game, or birth of a boy or girl. Ask each group to write down a few examples. For each example have the students indicate the number of possible outcomes, the chances of each of the outcomes occurring, and the percentage equivalency of each outcome. (For example in a bingo game there are 60 numbered balls. Therefore, the chance of any number being called is one out of sixty or 1/60. Expressed as a percentage it would be 16.6%.)

Ask the class if they have played games in which they have rolled number cubes to determine how many spaces to move. Ask how many possible outcomes there are and the percentage possibility for any one outcome. Ask them to predict how often a number would be rolled if there were three players and each rolled the number cube ten times. Ask if they have ever experienced the same number occurring more often than expected. Challenge the class to explain why this would occur. Ask the class to discuss in each group whether they think the outcome would be different if each player rolled 100 times. After discussing their ideas propose that a computer can be used to test out the idea that larger numbers in a trial will affect the outcome---closer to that of the theoretical outcome. This is called the Law of Large Numbers.

Introduce or review expected (or theoretical probability) and experimental (or actual probability) outcomes. For example, the theoretical outcome in coin tossing is 50% heads and 50% tails. If children flip a coin 10 times, the results may vary considerably from 50% for each possibility. Conversely, when the number of events is small, then there can be a wide variation between theoretical and experimental results. If the coin was flipped 1000 or 10,000 times the results will tend to get closer to 50%.

Ask the class what the chances would be of picking a blue M & M out of a bag of M & M's. If the students do not raise any questions before answering, challenge them to consider what variables might affect the outcome. Factors might include whether the M & M bag was new or the actual percentage of each color in a new bag. Did someone selectively pick out one color? If the bag was already partially eaten, what color M & M's had already been taken out? Are there as many blue as brown candies? Ask if one turn on a spinner has any effect on the chances of getting a particular result on the next spin. Ask if this is also true with the M & M's.

Propose that exploring how the number of trials of an event affects the experimental outcome might be quite time consuming and that a computer can be used to efficiently assist in the task. Direct the students to go to "Thinking About the Question".

After each group has had an opportunity to discuss the question in "Thinking About the Question" hold a brief discussion about the probability of a third child being a girl after the birth of three girl children. Next ask what their ideas about probability of the birth of a boy or girl. Ask for the probability expressed as a percentage. Ask for an explanation of how they arrived at their prediction. Ask for their ideas about the percentages of boy and girl births worldwide.

Propose that we can simulate an exploration of how the number of trials affects the difference between predicted and experimental outcome by flipping coins since the chances of a head or tail is also 50%. Indicate that they will use the spreadsheet program on their computer and a computer software program called "Winning Streak" to help collect, calculate, and represent this data. Direct the children to "Investigating the Question".

As the students work through the activity and answer the analysis questions, take short break to explore their mathematical reasoning. After each group has used the "Winning Streak" program to simulate 100 tosses, make a large class chart to record the results for at least 10 groups so that the class can record the results for 1000 trails.





Additional Teacher Background

The examples in this activity only include those in which the possible outcomes are not affected by outside variables. For example, we often speak of the chances of rain next day. Although there are only two possible outcomes, the probability of rain may not be 50%. Weather probabilities are affected by a complex set of variables. Predictions made by meteorologists are based on a vast amount of accumulated data regarding actual outcomes for repeated occurrences of a given complex of factors. The M & M example is different from the coin tossing because each trial effects the possibilities for the next by changing what is in the bag. Each coin toss has exactly the same possible outcome. Many people believe that if three heads were tossed in a row then the chances of a tail occurring on the fourth toss would be greater. This is a misunderstanding of probability.





The amount of time you spend on introductory discussions, data collection, and analysis, will determine your overall timeline. The following represents a possible timeline.

- One class period Introductory Discussion
- One class period Investigation I: Collecting experimental data
- One class period Investigation II: Collecting simulated data
- One class period Analysis

Additional days can be used for further investigations.

